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Asian Journal of Physics Vol. 23, No 3 (2014) 415-424

Novel zoom systems using a vortex pair

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Dedicate to Prof Kehar Singh

We describe the use of two refractive elements that have helical phase variations, here denoted as a vortex pair, for implementing varifocal lenses. Next, we employ vortex pairs for setting several nonconventional optical zoom systems. First, we discuss the Gaussian optics characteristics of single lens zoom system. Then, we describe a tunable afocal attachment, which is later employed for implementing a telephoto system whose elements are at fixed positions. Finally, we discuss a nonconventional zoom system, and we unveil a confocal device that scans axially a volumetric sample.

1 Introduction

For several applications in optics, it is convenient to modify the optical power of an imaging system. In a varifocal optical system, if one moves axially some elements of the device, then one can change continuously the focal length. Furthermore, in a zoom system, one can change the optical power, while preserving the positions of one pair of conjugate planes [1]. And in 2-conjugate zoom systems, one is able to preserve the positions of two independent conjugate pairs, while changing the optical power [2].

A different type of varifocal lenses was presented by Kitajima, several years ago [3, 4]. He proposed the use of two free-form elements working as a pair. If one displaces laterally the elements of the pair, then one can change continuously the optical power. This optical technique was forgotten for several decades, and it was rediscovered independently and simultaneously by Alvarez [5] and Lohmann [6-8].

Recently, we have revisited the Alvarez-Lohmann proposal for implementing several novel focalizers and some nonconventional apodizers by using optical elements that have vortex like complex amplitude variations [9, 10].

Here our aim is threefold. First, we revisit the use of a pair of free-form refractive elements, which is here denoted as a vortex pair, for optically implementing varifocal lenses. Second, we discuss the Gaussian optics characteristics of several zoom systems, which employ one or two vortex pairs as varifocal devices [11]. Third, we present a device that uses two vortex pairs for scanning axially a volumetric sample [12, 13].

To our end, in section 2, we revisit the use of two refractive elements, for setting a vortex pair, as a varifocal lens. In section 3, we show that a vortex pair is useful for setting a single-lens zoom system. In section 4, we describe the use of two vortex pairs for setting an afocal attachment, which is applied for setting a telephoto system that uses lenses at fixed positions. In section 5, we employ the afocal attachment for proposing a nonconventional zoom system. Next, in section 6, we present an optical device that scans axially, in a confocal fashion, a volumetric sample. And in section 7, we summarize our contributions.

2 Varifocal lenses using vortex phase delays

For clarifying our notation, we employ the optical processor that is depicted in Fig 1. At the Fraunhofer plane we place two refractive elements, which work as a pair. For the first element, the complex amplitude transmittance is

$$T_1(\rho, \varphi) = \exp\left\{ia\,\varphi\left(\frac{\rho}{\Omega}\right)^2\right\} circ\left(\frac{\rho}{\Omega}\right) \quad (1)$$

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In Eq (1) we include a constant phase factor that sets the maximum phase delay to a value of $27\pi a$. The Greek letters ρ and φ denote, respectively, the radial spatial frequency and the polar angle on the pupil aperture. The maximum value of ρ is Ω, which denotes the cut-off spatial frequency of the pupil aperture. Hence, the circular pupil aperture is represented by the circle function circ(ρ/Ω).

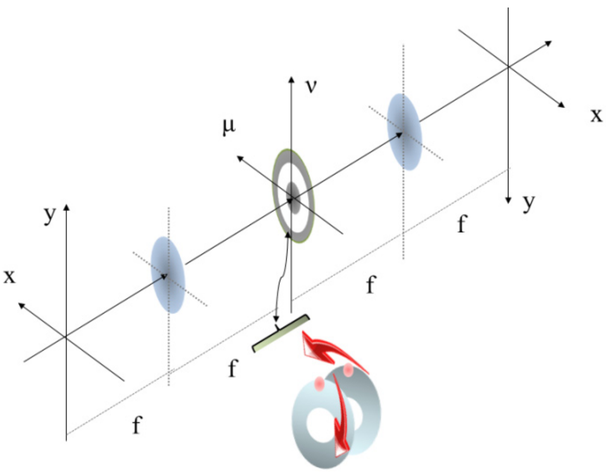

Fig 1. Optical setup depicting the location of a vortex pair at the Fraunhofer plane

Now, just behind the first refractive element, we place a second refractive element whose complex amplitude transmittance is the complex conjugate of the amplitude in Eq (1), That is,

$$T_2\,(\rho,\,\varphi)\ \exp\{ia\varphi\,(\rho/\Omega)^2\}\mathrm{circ}(\rho/\Omega). \tag{2}$$

In Fig 2 we show the phase variations and the interferogram of a quadratic phase profile that has helical variations, for $a = 10$.

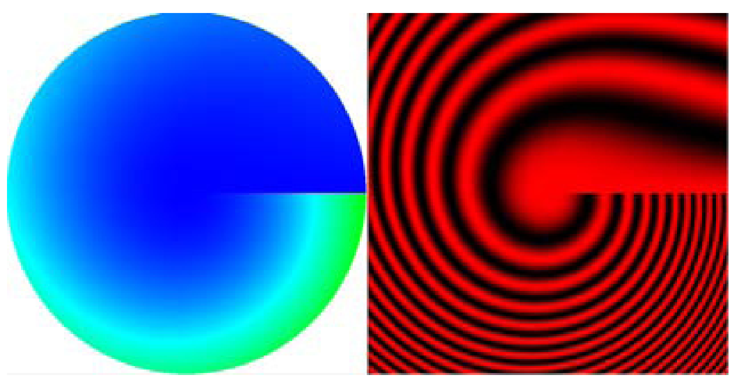

Fig 2. Phase delays and associated interferogram of a wavefront with radial and helical variations

The interferogram is obtained if one uses a plane wave as a reference beam and the complex amplitude in Eq (2) as the test beam. After we have introduced an in-plane rotation (say by an angle β) between the previously defined refractive elements, the overall complex amplitude transmittance is

$$P_{lens}\,(\rho,\,\varphi;\,\beta) = T_1\left(\rho,\,\varphi + \frac{\beta}{2}\right) T_2\left(\rho,\,\varphi - \frac{\beta}{2}\right) = \exp\left\{-i\,(\alpha\beta)\left(\frac{\rho}{\Omega}\right)^2\right\}\,circ\left(\frac{\rho}{\Omega}\right). \tag{3}$$

It is apparent from Eq (3) that the overall complex amplitude transmittance is independent of the polar angle φ. Furthermore, we note that by selecting the value of the angle β, one can control the maximum value of the quadratic phase delay. As is shown in the Appendix, the complex amplitude transmittance in Eq (3) is equivalent to the complex amplitude transmittance of a varifocal lens

$$P_{lens}\,(r,\,\varphi;\,\beta)\,\exp\{-i[(\beta/\lambda f\,)]r^2\}\mathrm{circ}(\rho/\Omega) \tag{4}$$

In Eq (4) we denote as r the radial variable over the pupil aperture. Its maximum value is R. We define as a vortex pair the complex amplitude transmittance either in Eq (3) or in Eq (4). In what follows we discuss the use of vortex pairs for setting several nonconventional zoom systems.

3 Single lens zoom system and telephoto attachments

For designing a single lens system it is convenient to define first the throw of the device, as the longitudinal distance between the input plane and the output plane. See for example reference [11]. That is, $T = z' - z$, where the letter z is the distance from the lens to the input plane, and z' is the distance from the lens to the output plane; as is depicted at the top of in Fig 3.

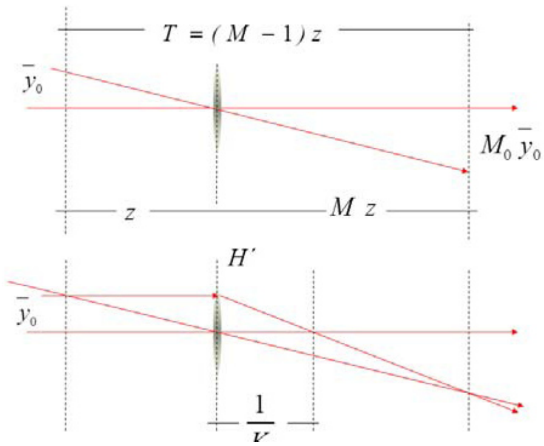

Fig 3. Single-lens zoom systems with variable magnification and fixed throw

The next relevant variable is the magnification, $M = z'/z$. The values of T and M are sufficient for specifying the value of z, as well as the optical power K of the lens; as is depicted at the bottom of Fig 3. In mathematical terms,

$$z = \frac{T}{M - 1} \tag{5}$$

$$K = -\frac{(1 - M)^2}{MT}. \tag{6}$$

For a single lens zoom system the selection of K is the last step, which is implemented by introducing an in-plane rotation between the elements of the vortex pair. Here, it is relevant to emphasize that the single lens zoom system works for a fixed value of T and for a wide range of values of M. To that end, one needs to move the vortex pair at the proper position z. After one places the vortex pair at the proper position z, then one sets the desired optical power by selecting the angle β.

4 A tunable afocal device and telephoto with lenses at a fixed positions

The afocal system that is shown in Fig 4 is a Galilean telescope, which uses a positive lens at the front of the optical system, and a negative lens at the back of the same device. Here, the first lens is a vortex pair, which has variable optical power

$$K_1 = qK_s; \quad 0 \le q < Q. \tag{7}$$

In Eq (7) the Latin letter q is a positive real number that is less than the real number Q; this latter number specifies the separation between the two vortex pairs as follows

$$d = \frac{1}{AK_s} \tag{8}$$

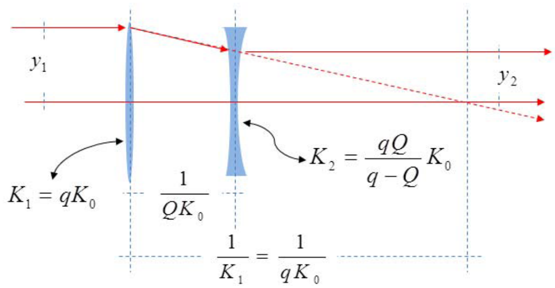

Fig 4. Afocal device employing two vortex pairs

In Eqs (7) and (8) the lower case letter s denotes the optical power of the substrate lens that supports the vortex pair. For implementing the afocal device the second lens must have the following optical power

$$K_2 \left(\frac{1}{Q} - \frac{1}{q}\right)^{-1} K_s; \quad 0 \leq q < Q. \tag{9}$$

Hence, for this device the magnification is

$$M = -\frac{K_1}{K_2} = 1 - \frac{q}{Q}; \quad 0 \leq q < Q. \tag{10}$$

It is apparent from Eq (10) that by using two vortex pairs one can reduce the width of a parallel beam from unity to very small values. In Fig 5 we plot the magnification as a function of the parameter q.

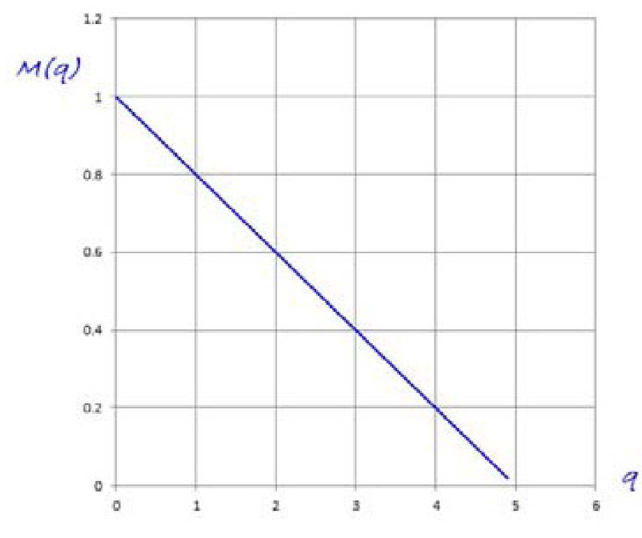

Fig 5. Magnification as a function of the parameter q

The magnification is independent of the optical power associated to the substrate lens supporting the vortex elements.

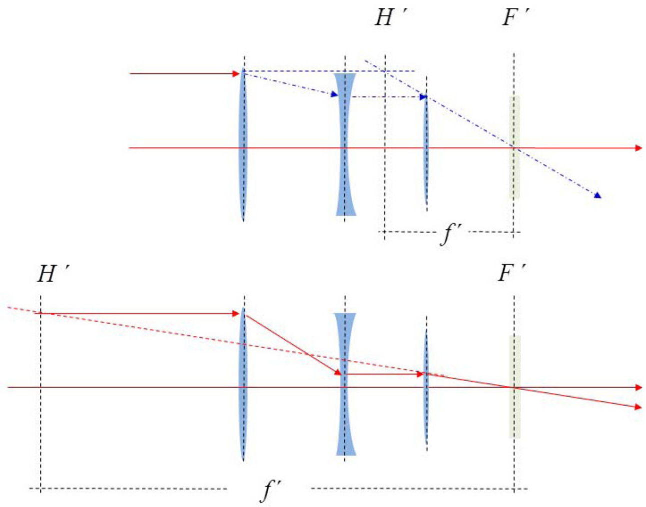

Fig 6. Telephoto system with lenses located at fixed positions

We recognize next that by using a positive lens behind an afocal device, one can design a telephoto system that uses lenses at fixed positions; as is depicted in Fig 6. As it was previously discussed when describing the afocal attachment, the focal lengths of the two vortex pairs are respectively,

$$f_1 = \frac{f_s}{q}; f_2 = \left(\frac{1}{Q} - \frac{1}{q}\right) f_s \tag{11}$$

It is straightforward to show that for the telephoto system in Fig 6, the overall focal length is

$$f' = \frac{Q}{Q - q} f_3. \tag{12}$$

In Eq (12) we denote as f_3 the fixed focal length of the last lens in the optical system. It is apparent from Eq (12) that the overall focal length changes from the value f_3 to the value $Q\,f_3$; which is obtained if $q = Q - 1$.

5 Nonconventional zoom systems

For setting a zoom system that has vortex pairs, we start by using the telecentric configuration in Fig 7. The optical setup uses two positive lenses, with focal lengths f_0 and f_3, respectively. Trivially, the magnification is

$$M_0 = -\frac{f_3}{f_0} \tag{13}$$

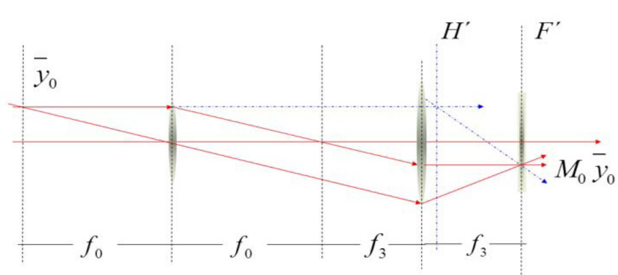

Fig 7. First stages for designing a nonconventional zoom system

Next as is depicted in Figure 8, between the elements of the optical system in Fig 7, we insert the afocal device previously discussed. We recognize that the device (in Fig 8) preserves the position of the input plane, as well as the position of exit plane. Consequently, the optical system has a constant throw $T = 2\,(f_0 + f_3)$. By tracing paraxial rays, it is easy to show that the magnification is

$$M = -\frac{Q}{Q-q} M_0; \quad 0 \leq q < Q. \tag{14}$$

In other words, for the optical system in Fig 8 the zoom range is

$$R = -\frac{Q}{q-Q}; \quad 0 \leq q < Q. \tag{15}$$

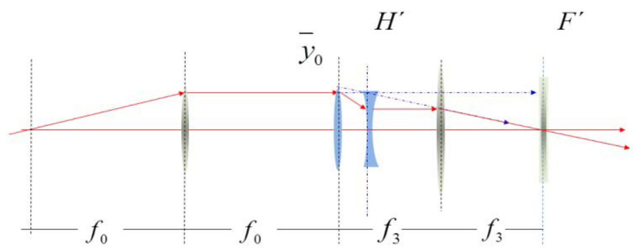

Fig 8. Nonconventional optical zoom

In Fig 9 we plot the zoom range as a function of the parameter q, for $Q = 12$ and for $0 \leq q \leq 11.8$.

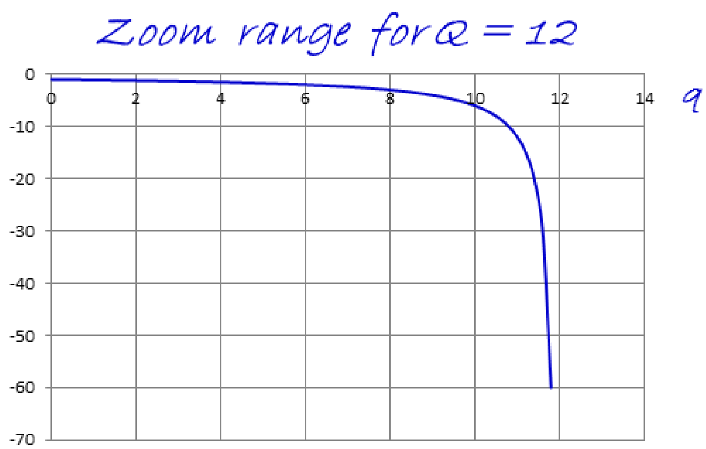

Fig 9. Zoom range as a function of the parameter q

In what follows we employ a different configuration for designing a nonconventional axial scanner.

6 Tunable axial scanners

In Fig 10 we show the schematics of an optical processor that employs two vortex pairs for producing an intermediate focal spot, which moves along the optical axis. A first vortex pair is used to generate, at variable axial positions, an intermediate focal spot that illuminates a 3-D sample. Next, a second vortex pair is used for imaging the intermediate focal spot at the fixed output plane. Hence, the throw ($T = 4\,f_0$) remains invariant. For achieving the desired purpose, the first vortex pair has the following optical power

$$R = -\frac{Q}{q - Q}; \quad 0 \le q < Q \tag{16}$$

Employing the paraxial ray tracing formulas, it is easy to find that the focal spot is located at

$$R = -\frac{Q}{q - Q}; \quad 0 \le q < Q \tag{17}$$

after the first vortex pair. And since $z_2 = -(2f_0 - z_1)$, then the focal spot is located before the second vortex pair at the distance

$$z_2(\beta) = \frac{1 - 2\beta}{\beta} f_0 \tag{18}$$

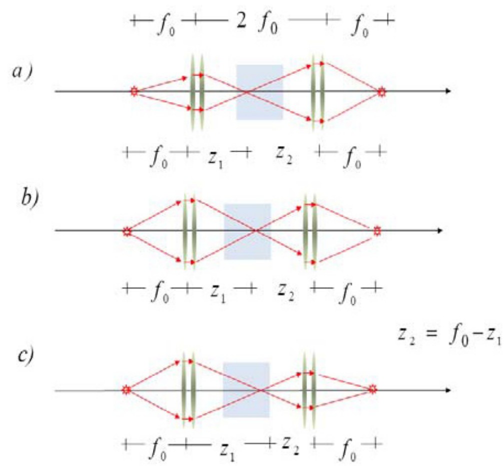

Fig 10. Confocal axial scanner: unfolded version

It is also easy to show that for the second pair the optical power is

$$K_2(\beta) = -\left(\frac{3\beta - 1}{2\beta - 1}\right)\frac{1}{f_0}. \tag{19}$$

In Fig 11 we plot Eqs (16) and (19) for the interval $0.5 < \beta < 5$.

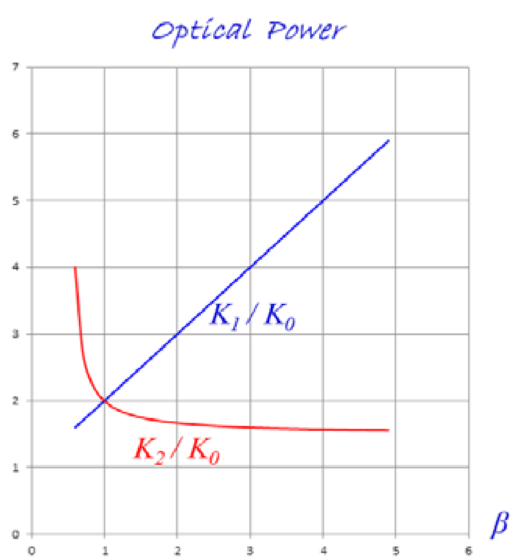

Fig 11. Optical powers as functions of the angle β

For this scanning device the total power is

$$K(\beta) = -\left(\frac{4\beta^2}{2\beta - 1}\right) K_0. \qquad (20)$$

In Fig 12 we plot Eq (20) and the focal length for the interval $0.5 < \beta < 5$.

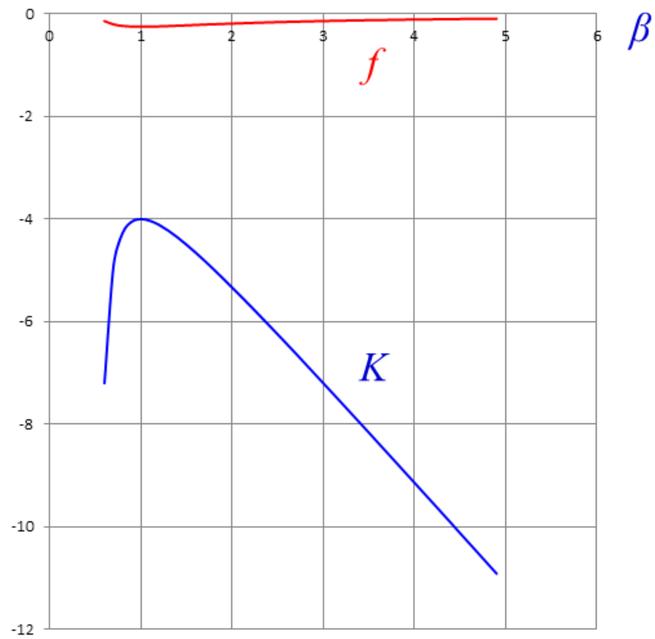

Fig 12. Optical power and focal length as functions of the angle β

Still related to the proposals of Minsky [12] and of Frosch and Korth [13], in Fig 13 we show a variant of the axial scanner. This is a folded version of the setup in Fig 10. By changing the focal length, f_z, of the vortex pairs one can select axial positions on a 3-D sample. The specific characteristics of this device are beyond our present scope.

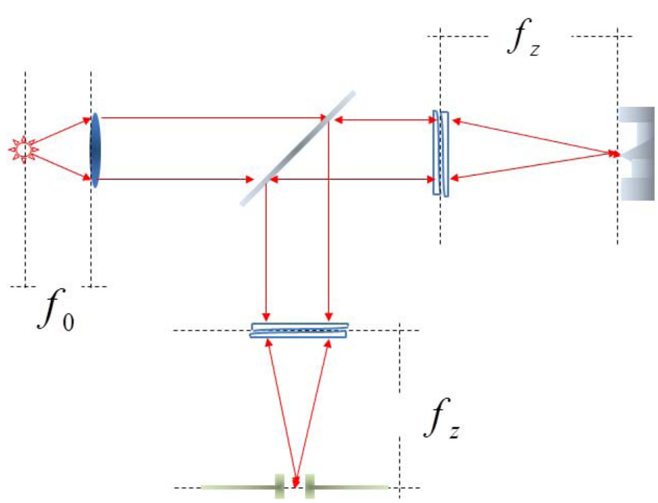

Fig 13. Confocal axial scanner: folded version

This is a folded version that uses two vortex pairs for selecting points on a sample, by changing the focal length, f_z, of the vortex pairs. The specific characteristics of this device are beyond our present scope.

7 Conclusions

We have indicated that a vortex pair is characterized by the use of two complex amplitude transmittances, which have helical phase variations. We have shown that a vortex pair is useful for controlling the optical power of lens, by introducing an in-plane rotation between the elements of the pair. And in this manner, one can implement a varifocal lens. We have described the Gaussian optics characteristics of a single-lens zoom system. Furthermore, we have shown that by using two vortex lenses, one can implement a tunable afocal attachment. Next, we have indicated that the afocal attachment is useful for setting a telephoto system with variable magnification despite the fact that the device has elements at fixed longitudinal positions. We have shown that the discussed afocal attachment is also useful for implementing an optical device that changes magnification, while preserving the locations of both the input plane and the output plane. Finally, we have unveiled a confocal device that scans axially volumetric samples, while preserving the locations of the initial point source, as well as that of the detector plane. Our present discussion is only valid in the paraxial regime, and if one uses monochromatic illumination.

Appendix

In the main text we use the mathematical expression

$$P_{lens}(\rho, \varphi, \beta) = \exp\left\{-i(a\beta)\left(\frac{\rho}{\Omega}\right)^2\right\} circ\left(\frac{\rho}{\Omega}\right) \quad \text{(A1)}$$

In Eq (A1) the radial spatial frequencies ρ and Ω are obtained using the following Cartesian variables

$$\rho = \frac{r}{\lambda f},\ \Omega = \frac{R}{\lambda f} = \frac{\rho}{\Omega} = \frac{r}{R}. \quad \text{(A2)}$$

Next we note that the optical path difference, denoted here with the Latin letter "*a*", is

$$a = (N - 1)\,\frac{t}{\lambda}. \quad \text{(A3)}$$

In Eq (A3) the letter N is the value of the refractive index of the material that is used for building the vortex element. The letter "t" denotes the maximum thickness of the refractive element; and the Greek letter lambda represents the wavelength of the optical radiation. Hence, the complex amplitude transmittance in Eq. (A1) is equivalent to

$$\begin{aligned} P_{lens}(\rho, \varphi, \beta) &= \exp\ \{-i[(N-1)\left(\frac{t}{\lambda f^2}\right)\beta]r^2\}\ circ\left(\frac{\rho}{\Omega}\right) \\ &= \exp\ \{-i[\left(\frac{t}{\lambda f}\right)]r^2\}\ circ\left(\frac{\rho}{\Omega}\right) \end{aligned} \quad \text{(A4)}$$

In Eq (A4) we recognize that the expression associated to the variable optical power is

$$K(\beta) = \left(\frac{\beta}{f}\right)(N-1)\left(\frac{t}{R^2}\right)\beta. \quad \text{(A5)}$$

Thus, a vortex pair generates a lens with tunable optical power.

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[*Received*:1. 4. 2014 ; *accepted*:1. 6. 2014]